

GRID EMT

SIMULATION INITIATIVE

Stability Analysis of Simulation of Stiff Switched LTI Systems

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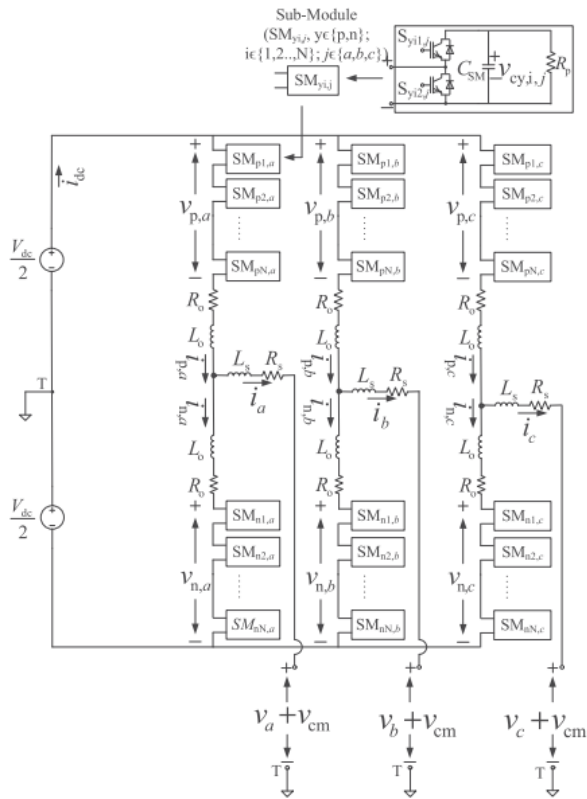
Outline

- (i) What is a stiff switched LTI system?
- (ii) Why is state partitioning important?
- (iii) Stability analysis of 2-D systems
- (iv) Examples

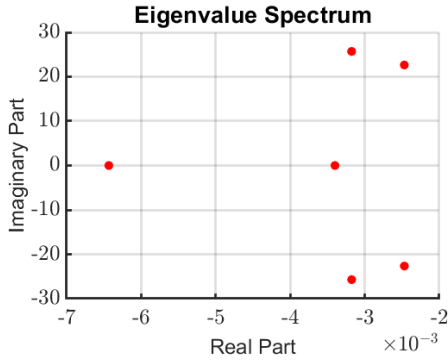


Stiff switched LTI system

Example: Modular Multilevel Converters (MMCs)

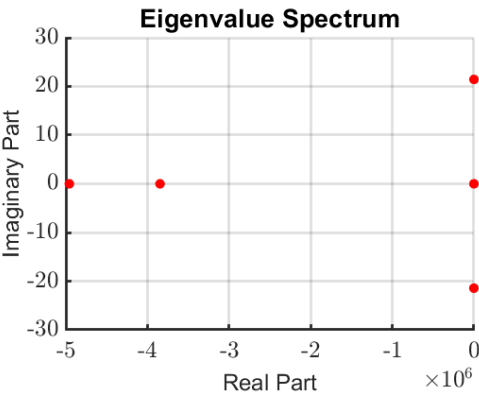


Switched linear DAEs with time-varying stiffness!

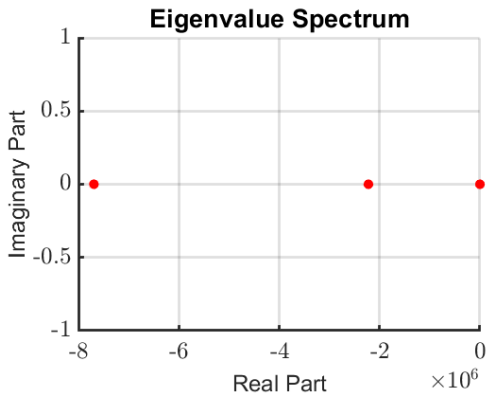


Blocked state with **no** open arm

Blocked state with **3** open arms



Blocked state with **all** open arms



Motivation for state partitioning

For discretizing all states in stiff systems with a given step size h (to achieve desired accuracy):

- Backward Euler (Implicit solver) works just fine: **computationally intensive!**
- Forward Euler (Explicit solver) needs step size $\ll h$: **numerical stability constraint induced by stiff states BUT computationally fast!**

Why not discretize stiff states using Implicit method and non-stiff states using Explicit method?

- Implicit-Explicit (ImEx) solver: **best of both worlds!**

Need to **partition** stiff states and non-stiff states of a system for a given h



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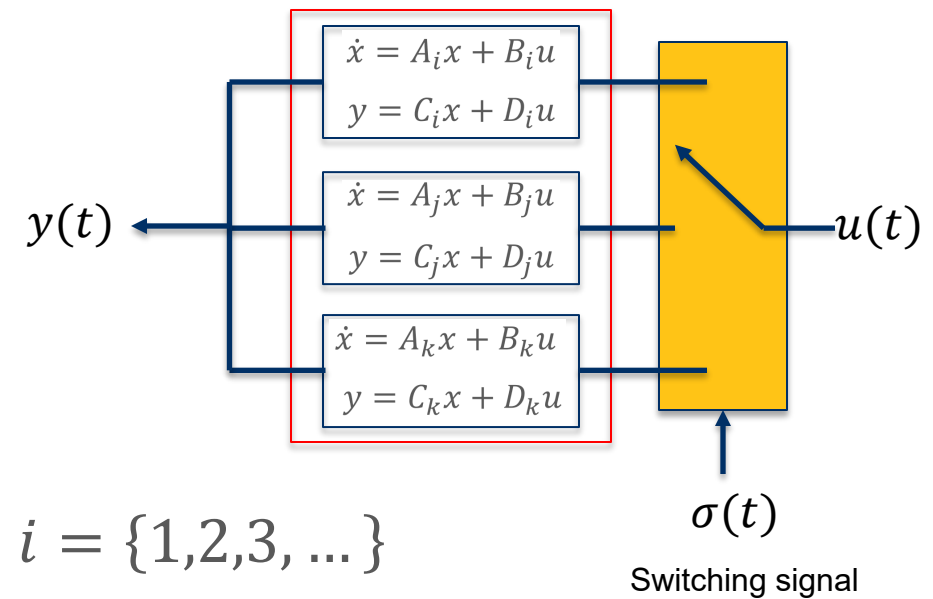
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System under consideration

- Consider a switched LTI system as:

$$\dot{x} = A_i x + B_i u$$

$$y = C_i x + D_i u \quad , \quad i = \{1, 2, 3, \dots\}$$



Assumption:

- The system is quadratically stable when $u = 0$
 $\Rightarrow$ The forced system is BIBO stable

Implication:

- The energy of the unforced system decreases monotonically regardless of the switching sequence!**



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Stability of Simulation

- Is it possible for the simulation of the unforced system to be quadratically stable?
- Or, is it possible for the simulation of the forced system to be BIBO stable?

⇒ When is a simulation quadratically stable ?



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Simulation using one step methods

- Discrete time simulation model (with $u = 0$) :

$$x_{n+1} = G_i x_n$$

- Examples

- Forward Euler (explicit) method: $G_i = I + hA_i$
- Backward Euler (implicit) method: $G_i = (I - hA_i)^{-1}$

⇒ When is a simulation quadratically stable ?



$$G_i^T V G_i - V < 0, \quad V > 0$$



Some facts!

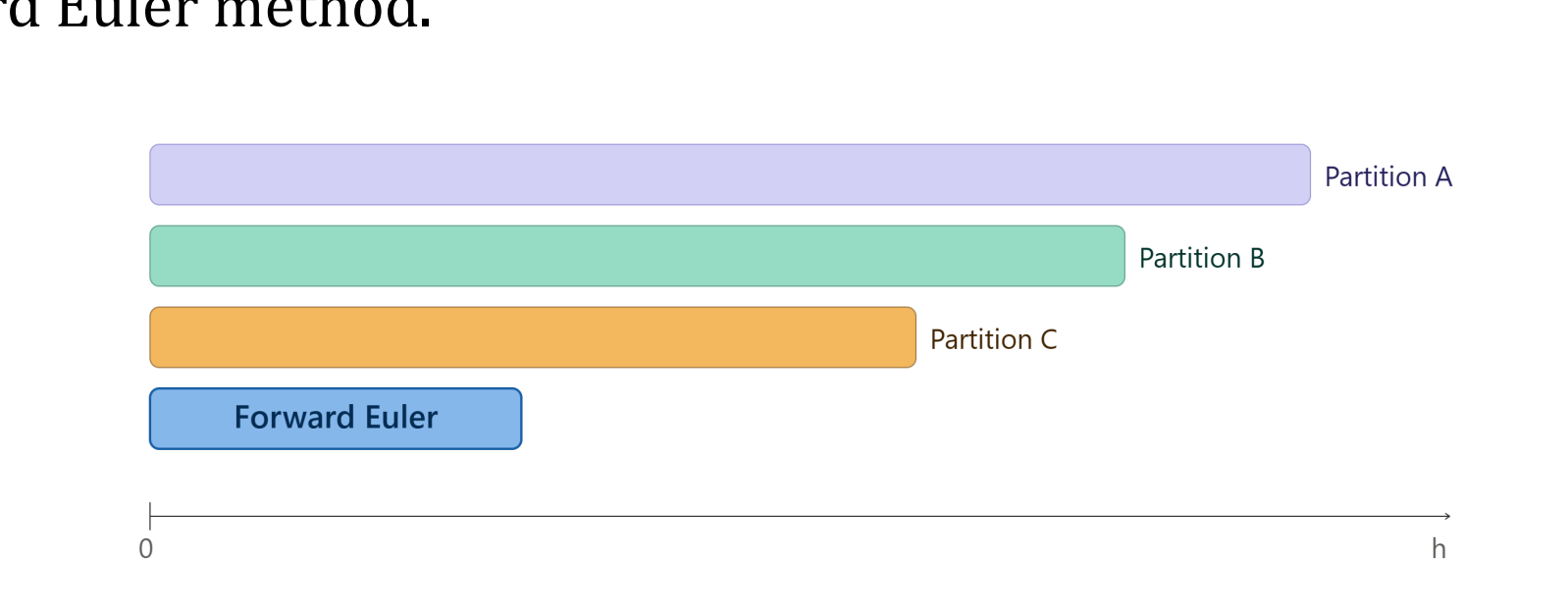
- Backward Euler method-based simulation is stable $\forall h > 0$!
- Forward Euler method-based simulation is **NOT** !
- What can we say about the stability of simulation conducted using a hybrid partition of the system's states under a given h ?



Our findings so far!

- **Case 1:** All A_i 's are diagonal

Every partition has a quadratic stability domain **at least as large** as that of the forward Euler method.



Our findings so far!

- **Case 2:** All A_i 's (in 2-D) are **lower triangular** and $(A_i)_{22} < (A_i)_{11} < 0$
- Example: $A_i = \begin{bmatrix} -2 & 0 \\ (A_i)_{21} & -3 \end{bmatrix}$

Suppose x_1 discretized explicitly and x_2 implicitly. If we can find $W_1 > 0, W_2 > 0$ s. t.

$$\frac{W_2}{W_1} < \underbrace{-2 \left(1 - \frac{(A_i)_{22}^2}{((A_i)_{22} - 2(A_i)_{11})^2} \right) \left(\frac{(A_i)_{22}}{(A_i)_{21}} \right)^2}_{>0}$$

for all i , then this partition has a quadratic (as well as BIBO) stability domain **larger** than the forward Euler method.

Such W_1, W_2 always exist!



Our findings so far!

- **Case 3:** All A_i 's (in 2-D) are **upper triangular** and $(A_i)_{22} < (A_i)_{11} < 0$
- Example: $A_i = \begin{bmatrix} -2 & (A_i)_{12} \\ 0 & -3 \end{bmatrix}$

Suppose x_1 discretized explicitly and x_2 implicitly. If we can find $W_1 > 0, W_2 > 0$ s.t.

$$\frac{W_1}{W_2} < \underbrace{-8 \left(1 - \frac{(A_i)_{22}^2}{((A_i)_{22} - 2(A_i)_{11})^2} \right)}_{>0} \left(\frac{\left(1 - \frac{2(A_i)_{11}}{(A_i)_{22}} \right)^2}{6 \frac{(A_i)_{12}}{(A_i)_{22}}} \right)^2$$

for all i , then such a partition has a quadratic (as well as BIBO) stability domain **larger** than that of the forward Euler method.

Such W_1, W_2 always exist!



Our findings so far!

- **Case 4:** All A_i 's (in 2-D) satisfy $(A_i)_{12}(A_i)_{21} > 0$ and $(A_i)_{22} < (A_i)_{11} < 0$
- Example: $A_i = \begin{bmatrix} -2 & 1 \\ 2 & -3 \end{bmatrix}$

Suppose x_1 discretized explicitly and x_2 implicitly. If we can find $W_1 > 0, W_2 > 0$ s.t.

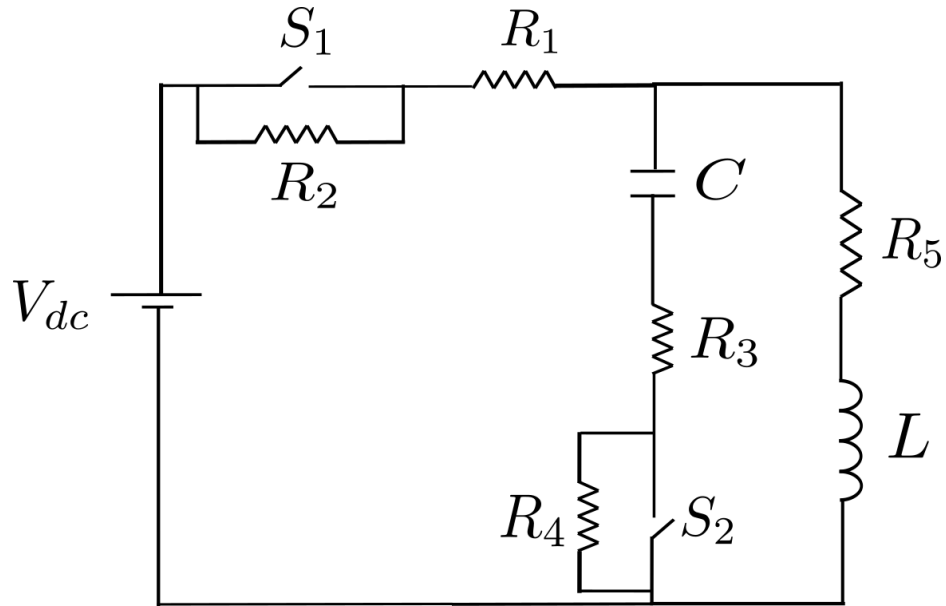
$$\left(\frac{W_1}{W_2}\right)^2 \left(\underbrace{\frac{-36(A_i)_{12}^2}{(A_i)_{22} - 2(A_i)_{11}^2}}_{<0} - \frac{4(A_i)_{21}^2}{(A_i)_{22}^2} \right) - \frac{W_1}{W_2} \left(\frac{16(A_i)_{12}^2(A_i)_{21}^2}{(A_i)_{22}^2((A_i)_{22} - 2(A_i)_{11})^2} + \frac{8(A_i)_{12}(A_i)_{21} - 8(A_i)_{22}^2}{((A_i)_{22} - 2(A_i)_{11})^2} + 8 \right) > 0$$

for all i , then such a partition has a quadratic (as well as BIBO) stability domain **larger** than that of the forward Euler method.



Results

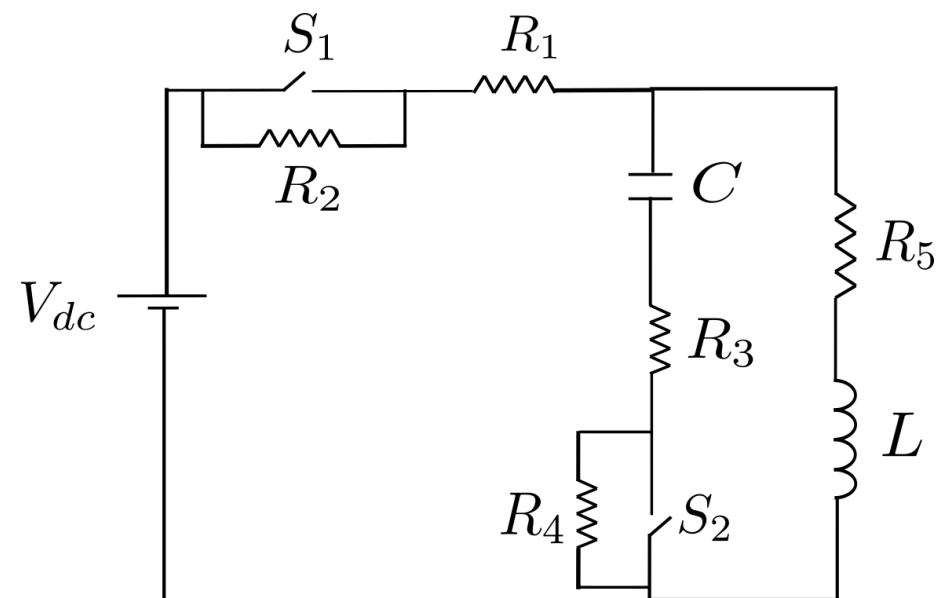
- Example 1: A toy switched RLC circuit



- 2 switches
- 4 switching states
- 2 states
 - inductor current (x_1)
 - capacitor voltage (x_2)



Results



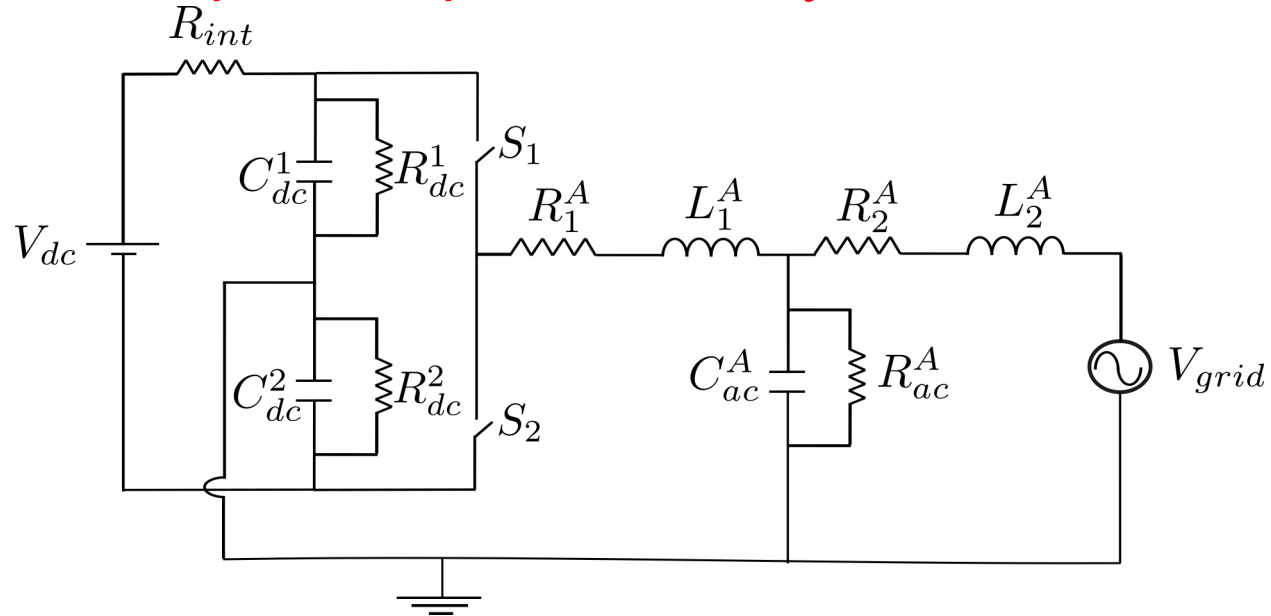
L	C	R_1	R_2	R_3	R_4	R_5
0.3e-3	3.9e-3	0.1e-3	0.5e-1	1e-2	1	1e-3

h (μ s)	Output
0.1	$S_E = \{x_1, x_2\}, S_I = \Phi$
0.8	$S_E = \{x_2\}, S_I = \{x_1\}$
1	$S_E = \{x_2\}, S_I = \{x_1\}$
5	$S_E = \Phi, S_I = \{x_1, x_2\}$
10	$S_E = \Phi, S_I = \{x_1, x_2\}$

Under a higher h , only the partition(s) with a greater number of implicitly discretized states are BIBO stable!

Results

- Example 2: 1-phase 2LC system



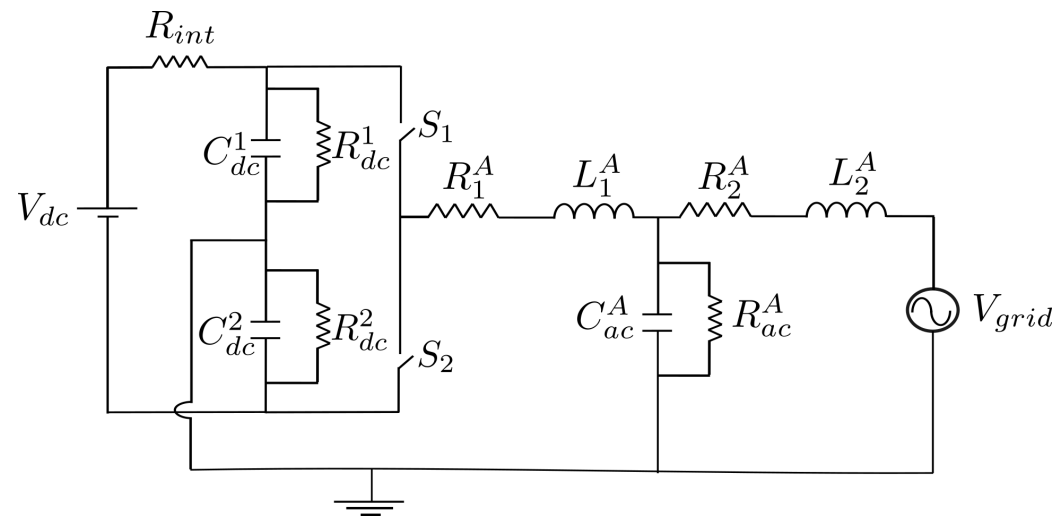
- 2 switches
- 2 switching states
- 5 states
 - 2 dc side capacitor voltages (x_1, x_2)
 - inductor current at converter's output (x_3)
 - inductor current at grid's input (x_4)
 - ac side capacitor voltage (x_5)



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Results



R_{int}	C_{dc}^1	C_{dc}^2	R_{dc}^1	R_{dc}^2	R_1^A	L_1^A	R_2^A	L_2^A	C_{ac}^A	R_{ac}^A
1e-3	3.9e-3	3.9e-3	20e3	20e3	1e-3	0.3e-3	1e-3	0.125e-3	0.2e-3	20e3

h (μ s)	Output
0.8	$S_E = \{x_1, x_2, x_3, x_4\}, S_I = \{x_5\}$
1	$S_E = \{x_1, x_2, x_3, x_4\}, S_I = \{x_5\}$
3	$S_E = \{x_1, x_2\}, S_I = \{x_3, x_4, x_5\}$
5	$S_E = \Phi, S_I = \{x_1, x_2, x_3, x_4, x_5\}$
10	$S_E = \Phi, S_I = \{x_1, x_2, x_3, x_4, x_5\}$

Under a higher h , only the partition(s) with a greater number of implicitly discretized states are BIBO stable!

Conclusions

- Quadratic stability and BIBO stability of hybrid discretization methods were analyzed specifically for 2-D systems
- Stability of several partitions of a toy switched RLC circuit and the 1-phase 2LC system were analyzed
- At a higher h , partitions with a greater number of implicit states were found stable



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Next Steps

- Evaluate the stability of hybrid partitions for high dimensional systems.
- Apply the stability tests on power electronic based switched systems.



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Thank You

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