

GRID EMT

SIMULATION INITIATIVE

Iterative Solvers for Accelerating EMT

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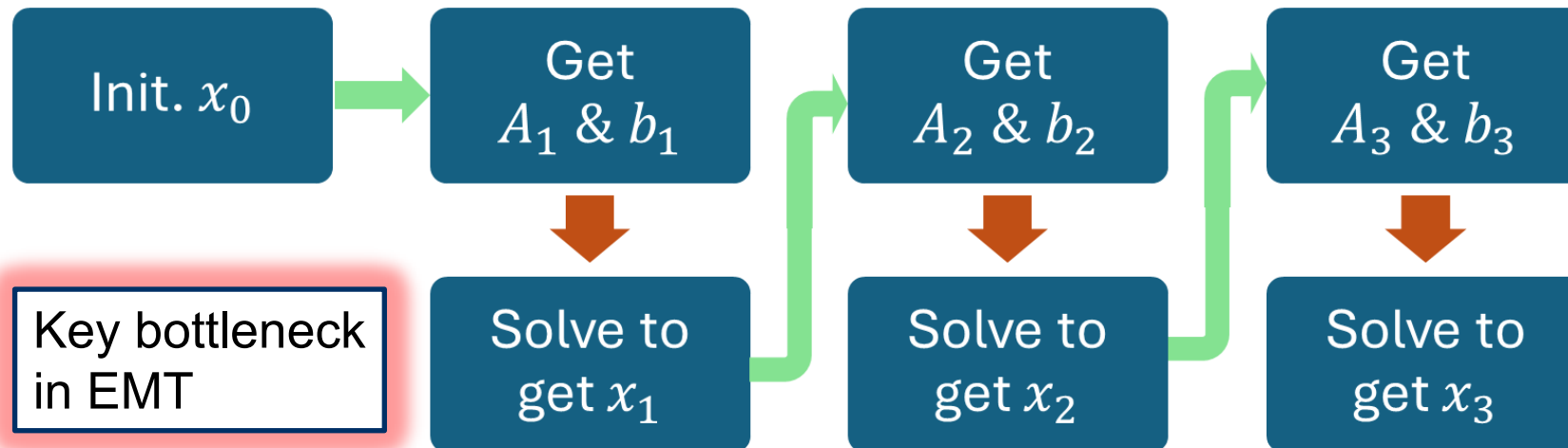
EMT Simulation – Core Computation Bottleneck

- Dynamics of power grid with EMT can be represented as a DAE

$$\begin{aligned} \dot{z} &= f(z, y, t), \\ 0 &= g(z, y, t), \end{aligned} \xrightarrow{\text{Discretize}} A_k x_k = b_k$$

$x_k = \begin{bmatrix} z_k \\ y_k \end{bmatrix}$ is the unknown state vector and contains both the differential and algebraic variables at time step k .

A_k is a large sparse matrix and depends on x_{k-1}
 b_k is the right-hand-side vector and depends on x_{k-1}



Approach to Solve Sparse Linear Equations I

- Factorize the sparse matrix $A_k = L_k U_k$. Equation is $L_k(U_k x_k) = b_k$ Solution is obtained through two sequential stages that are very simple to solve:
- Forward Substitution*: Solve $Ly = b$ for the intermediate vector y .
- Backward Substitution*: Solve $Ux = y$ for the final solution vector x .

$$\begin{bmatrix} 1 & & & & & & & & & \\ & 1 & & & & & & & & \\ & \ell_{32} & 1 & & & & & & & \\ & & & 1 & & & & & & \\ \ell_{51} & \ell_{52} & \ell_{53} & & 1 & & & & & \\ \ell_{61} & & \ell_{63} & & \ell_{65} & 1 & & & & \\ \ell_{71} & & & & \ell_{75} & \ell_{76} & 1 & & & \\ & & & & & & & 1 & & \\ & & & & & & & \ell_{98} & 1 & \\ & & & & & & & & \ell_{10,9} & 1 \\ & & & & \ell_{10,4} & & \ell_{10,6} & \ell_{10,7} & \ell_{10,8} & \ell_{10,9} & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \\ y_6 \\ y_7 \\ y_8 \\ y_9 \\ y_{10} \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \\ b_5 \\ b_6 \\ b_7 \\ b_8 \\ b_9 \\ b_{10} \end{bmatrix} \quad \Bigg| \quad \begin{bmatrix} u_{11} & & & & u_{15} & u_{16} & u_{17} \\ & u_{22} & u_{23} & & u_{25} & & \\ & & u_{33} & & u_{35} & u_{36} & \\ & & & u_{44} & & & \\ & & & & u_{55} & u_{56} & u_{57} \\ & & & & & u_{66} & u_{67} \\ & & & & & & u_{77} \\ & & & & & & & u_{88} & u_{89} & u_{8,10} \\ & & & & & & & & u_{99} & u_{9,10} \\ & & & & & & & & & u_{10,10} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \\ x_8 \\ x_9 \\ x_{10} \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \\ y_6 \\ y_7 \\ y_8 \\ y_9 \\ y_{10} \end{bmatrix}$$

Approach to Solve Sparse Linear Equations II

- A_k changes with time-step and thus recomputing L_k and U_k at every time step dominates the overall runtime. Note that the change in A_k is usually small.
- Can we leverage L_1 and U_1 to solve $A_k x_k = b_k$ for any k ?

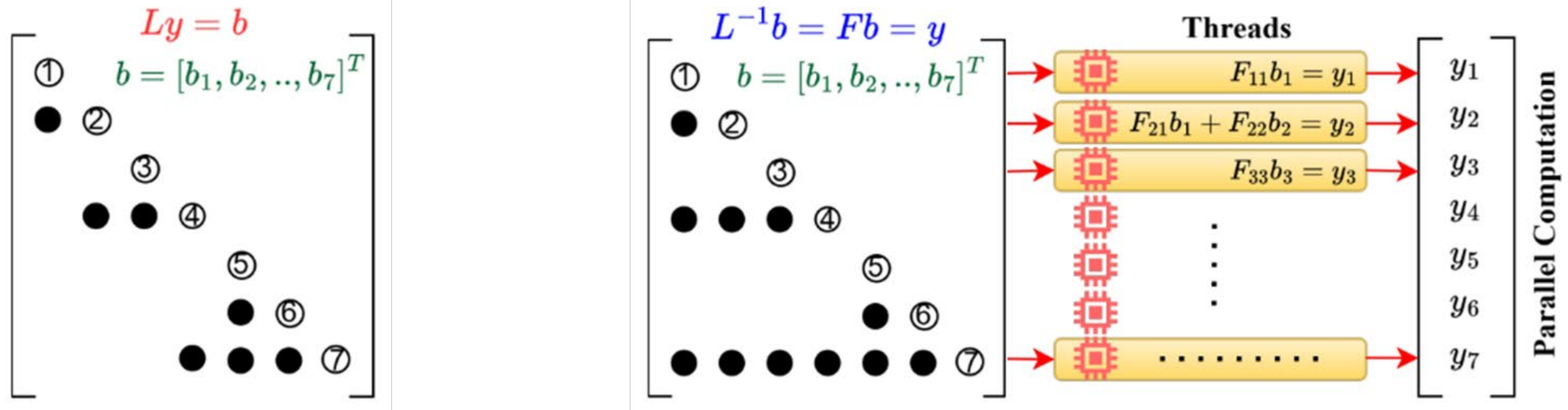
$$A_k x_k = b_k \Rightarrow A_1^{-1} A_k x_k = A_1^{-1} b_k \Rightarrow U_1^{-1} L_1^{-1} A_k x_k = U_1^{-1} L_1^{-1} b_k \Rightarrow \hat{A}_k x_k = \hat{b}_k$$

- The actions of L_1^{-1} and U_1^{-1} are realized through forward and backward substitutions using the stored LU factors.
- Iterative methods: Solve this “preconditioned” system from an initial guess, $\hat{x}_k^{(0)}$ and update $(\hat{x}_k^{(1)}, \hat{x}_k^{(2)}, \dots)$ using **matrix-vector multiplications** with $\hat{A}_k$ and $\hat{x}_k^{(i)}$.
- GMRES and BiCGSTAB are classic iterative methods



Approach to Solve Sparse Linear Equations III

- Forward and backward substitutions are sequential due to strong data dependencies and therefore offer limited parallelization potential



- The inverse factors considered here preserve sparsity and are computed exactly for the class of power-system matrices arising in EMT simulations



Results on IEEE 39 bus system with 940 states

- Total time for 100 time steps of EMT simulation with 1 micro second time step
- Tested on 24-core intel Xeon CPU with 256 GB DDR4 RAM

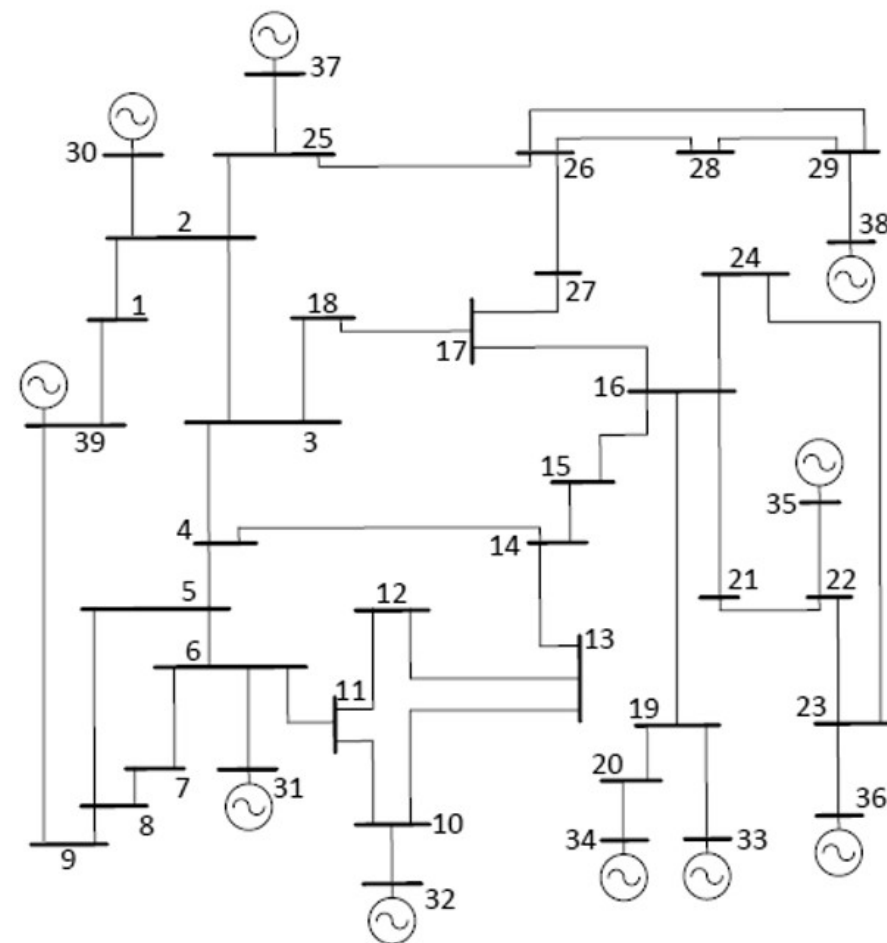
Method	Execution Time (ms)	Speedup vs. KLU	Error Norm
KLU (Circuit-Optimized Direct)	22.330	1.00	4.478×10^{-7}
GMRES (No Preconditioning)	525.376	< 0.1	3.873×10^{-3}
GMRES (Left Preconditioned)	10.086	2.32	4.478×10^{-7}
GMRES (Right Preconditioned)	5.352	4.25	4.478×10^{-7}
BiCGSTAB (Right Preconditioned)	4.198	5.41	4.478×10^{-7}
BiCGSTAB-SIF (Right Preconditioned)	3.281	7.5	4.478×10^{-7}



Results on IEEE 39 bus system with 940 states

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- 100 Time steps with $1\mu s$ time step

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Results on IEEE 39x20 bus system with 18800 states

- Replicated 39 bus system 20 times and considered this as a single system
- The A matrix has 50400 non-zeroes
- The lower acceleration is likely a result of not fully exploiting multi-core CPUs

Method	Linear Solve Time (ms)
KLU (Circuit-Optimized Direct)	509.298
GMRES (No Preconditioning)	N/A
GMRES (Left Preconditioned)	566.202
GMRES (Right Preconditioned)	209.528
BiCGSTAB (Right Preconditioned)	222.440
BiCGSTAB-SIF (Right Preconditioned)	205.271



Conclusions and Next Steps

- Exploiting the unique features of EMT, namely sparsity and temporal continuity, with iterative solvers can significantly outperform traditional sparse solvers.
- Preconditioning is necessary for achieving practical tolerance
- Compared to KLU, Right-preconditioned BiCGSTAB-SIF achieves
 - ~**7.5x** acceleration for the IEEE 39 bus system
 - ~**2x** acceleration the IEEE 39x20 bus system
- Next steps include
 - Test on a large-scale power system with switching dynamics
 - Leveraging parallelism on multi-core CPUs and GPUs to speed up the solver



Thank You

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