

GRID EMT

SIMULATION INITIATIVE

Advanced Stiffness Detection Algorithms for EMT Simulation

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Detecting stiffness and identifying its sources is critical to computational efficiency

There is not a precise definition of stiffness, instead various qualitative descriptions exist:

- The system contains a set of solution components that decay much more rapidly than others
- Stable time step sizes are much smaller than required to accurately represent the solution
- Neighboring solutions approach the solution of interest at a rate which is very large in comparison with the rate at which the solution varies

Stiffness depends on many factors: system dynamics, initial conditions, integration interval, desired accuracy, and properties of the numerical method

Selecting the right numerical method to advance a given system in time is essential to performance

**Stiff
Chemical
Kinetics
Problem**

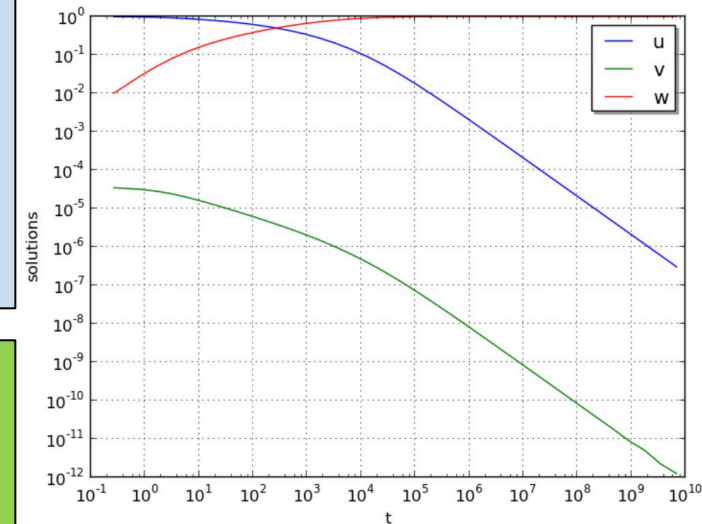
$$\begin{aligned}u' &= -.04 u + 10^4 v w \\v' &= .04 u - 10^4 v w - 3 \times 10^7 v^2 \\w' &= 3 \times 10^7 v^2\end{aligned}$$

Adams (non-stiff)

Steps: 15,635,771
Failed Steps: 28
Wall time: 2m 39s

BDF (stiff)

Steps: 542
Failed Steps: 22
Wall time: 0.8s



BDF ~200 × Faster

Examining the eigenvalues and eigenvectors of the linearized problem provides insights into system dynamics

Consider the linear ODE system

$$y' = Ay + \varphi(t), \quad y(t_0) = y_0$$

The general solution is given by

$$y(t) = \sum_{i=1}^N k_i e^{\lambda_i t} v_i + \psi(t)$$

- λ_i – eigenvalues of A
- v_i – right eigenvectors of A
- k_i – determined by the initial state
- $\psi(t)$ – a slow manifold solution

If $\text{Re } \lambda_i < 0$, then $e^{\lambda_i t} \rightarrow 0$ as $t \rightarrow \infty$

- Fast / slow transients: $k_i e^{\lambda_i t} v_i$
- Solution all states decay to: $\psi(t)$

Eigenvalues tell us the rate and frequency of transient dynamics

Eigenvectors tell us how strongly a variable is connected to a transient

With this information we can tailor the integration approach to the dynamics of the problem

For example, **implicit-explicit (ImEx)** methods apply an implicit method to the stiff dynamics and an explicit method to the non-stiff dynamics to alleviate the stability limitations from the fastest modes

Linear Stability Regions

Explicit
Euler

Implicit
Euler

Explicit Euler steps are restricted by $\max |\lambda_i|$ while Implicit Euler step are limited by accuracy

Participation factors provide an approach to classify variables as stiff or non-stiff for implicit-explicit partitioning

Computing Participation Factors

For the linear DAE $My' = Ay, \quad y(t_0) = y_0$

1. Solve the generalized eigenvalue problem

$$Av_i = \lambda_i Mv_i, \quad w_i^H A = \lambda_i w_i^H M$$

to get the eigenvalues λ_i and eigenvectors v_i (right) and w_i (left) for the *differential* variables

2. Compute the participation of mode i in variable k

$$p_{ki} = v_{ki} M^H w_{ki}$$

where $w_i^H M v_i = 1$

3. Normalize the scores

$$\hat{p}_{ki} = |p_{ki}| / \sum_l |p_{li}|$$

Weighting Modes and Scoring Variables

Given λ_i and $\hat{p}_{ki}$:

4. Choose a weighting scheme (W_i) to identify critical (stiff) modes of the problem e.g.,

- Largest magnitude: $|\lambda_i|$
- Fastest decay: $\max(0, -\text{Re}(\lambda_i))$
- Highest frequency: $\max(|\text{Im}(\lambda_i)|)$
- Stable step: $1/h_{i,\text{stab}}$

5. Score each variable by using the mode weight and normalized participation factor

$$s_k = \sum_i W_i \hat{p}_{ki}$$

Pérez-Arriaga, et al., 1982

Variable scoring and automatic method selection in a three phase RLC problem with RE-INTEGRATE

Linear DAE $My' = Ay + b(t)$ with 6 algebraic (V_1 and V_2) and 12 differential ($i_{L_2}, V_{C_3}, i_{L_4}, V_{C_4}$) states

RE-INTEGRATE Demo: Three phase RLC problem



Participation factors and variable scoring to identify stiff variables in RLC example

Differential mode weights ($|\lambda_i|$)

Constraints ($\lambda_i = \infty$) are excluded

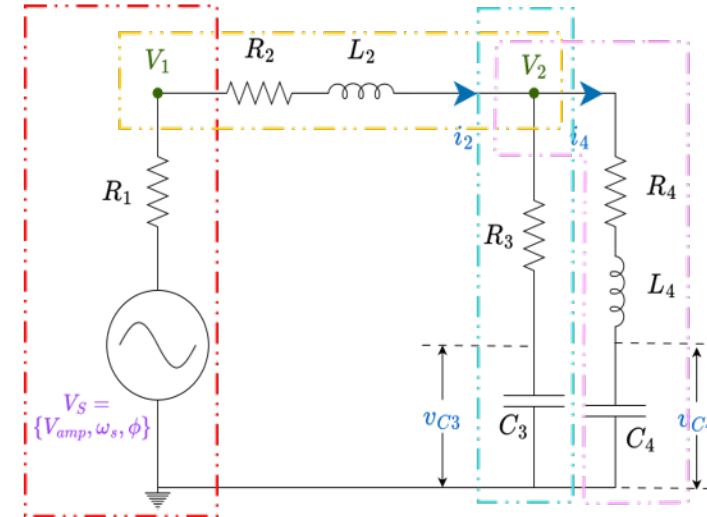
$$\lambda_{1,9,12} \approx -4 \times 10^6, \lambda_{2,10,13} \approx -53 + 589i, \lambda_{3,11,14} \approx -12$$

Algebraic variables are always treated implicitly

differential variable participation factors, lighter colors indicates a stronger association with a mode

Variables with higher participation in a strongly weighted mode are given a higher score indicating stiffness

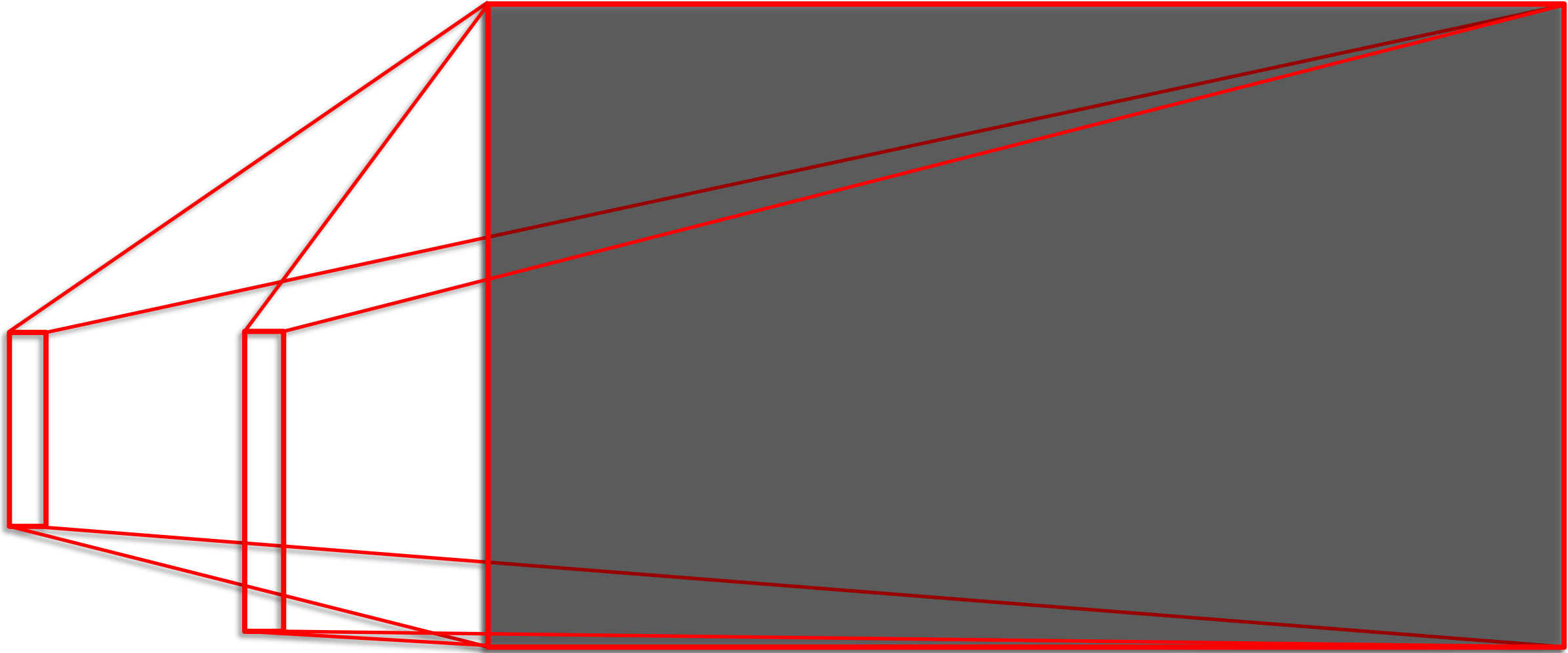
In this case, the i_{L_2} variables are the stiffest followed by the i_{L_4} variables



$V_{1,a}$
 $V_{1,b}$
 $V_{1,c}$
 $V_{2,a}$
 $V_{2,b}$
 $V_{2,c}$
 $i_{L_2,a}$
 $i_{L_2,b}$
 $i_{L_2,c}$

 $i_{L_4,a}$
 $i_{L_4,b}$
 $i_{L_4,c}$

Evaluating participation factor scoring in nonlinear systems with variable stiffness over time



Summary and Next Steps

- Stability limited (e.g., explicit) methods struggle to efficiently evolve stiff systems, requiring significantly more work than more stable (e.g., implicit) approaches to accurately resolve the solution
- As such, identifying which parts of a system contribute most to stiffness is critical to efficiency
- Participation factors offer an approach to relate stiff modes with specific variables to inform splitting choices in implicit-explicit methods
- This approach has been incorporated into RE-INTEGRATE for automatic method selection and correctly identifies stiff variables in linear test problems
- We are currently evaluating effectiveness and computational cost of the method on nonlinear systems where stiffness varies over time
- Next steps will focus on extending the approach to large sparse using a sub-system approach or utilizing iterative solvers to target a subset of eigenvalues and eigenvectors for scoring
- We are also investigating alternative stiffness detection approaches using multiple embedded error estimates to detect when step sizes are limited by method stability rather than solution accuracy

Thank You

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